Transition from the macrospin to chaotic behaviour by a spin-torque driven magnetization precession of a square nanoelement

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We demonstrate (using full-scale micromagnetic simulations) that the spin injection driven steady-state precession of a thin magnetic nanoelement exhibit a complicate transition from the quasi-macrospin to the chaotic behaviour with the increasing element size. For nanoelement parameters typical for those used experimentally we have found that the macrospin approximation becomes invalid already for very small nanoelement sizes (~ 30 nm), in contrast to the previously reported results (Li and Zhang, Phys. Rev., **B68**, 024404-1 (2003)).

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Magnetic excitations induced in a thin layer by a spin-polarized current injection (first predicted theoretically [1] and soon confirmed experimentally [2]) are at present one of the most intensively studied magnetic phenomena due to their very interesting physical nature and highly promising potential applications e.g., for fast switching of nanoelements [3] and design of nanosized *dc*-current driven microwave generators [4]. Due to the complicate remagnetization processes involved it was realized very quickly [5] that full-scale micromagnetic simulations should be carried out to support corresponding experiments, because simulations in a macrospin approximation [6], being a necessary first step in understanding some basic physics, can not explain many important features of experimental observations [3, 4].

One of the most relevant problems which could be answered by such simulations is the determination of the critical nanoelement size for the transition from a single- to a multi-domain behaviour during the switching/precession process. This question was addressed in one of the first papers where full-scale micromagnetic simulations of the spin-transfer induced remagnetization were carried out [7]. It was claimed in [7] that a nanoelement with magnetic parameters and thickness typical for real experiments remains virtually single-domain for the lateral size as large as 64×64 nm up to the highest spin torque tested in the simulations (and achievable experimentally).

In this paper we present a systematic study of magnetization structures occurring in square nanoelements of various sizes during the so called steady-state precession [6]. We have found that the transition from a quasi-macrospin precession to a multi-domain state occurring by the increase of a nanoelement size is very complicate; it involves several bifurcations and ends up with a fully chaotic system behaviour. According to our simulations (for parameters equal to those explicitly specified in [7]) the transition to a multi-domain configuration occurs in fact already for the size $\approx 35 - 40$ nm (the difference between our results and those reported in [7] is probably due to a much too high exchange constant used by Li and Zhang [10]).

In our simulations of the steady-state precessional states we have used the following system: a square-shaped monolayer element with the thickness $d = 2.5$ nm, saturation magnetization $M_S = 950$ G, uniaxial anisotropy with the anisotropy field $H_K = 500$ Oe along the $0x$-axis (all these parameters are identical to those used in [7]) and the exchange constant $A = 2 \times 10^{-6}$ erg/cm. The lateral size of a nanoelement was varied from 16×16 (where the element was clearly single-domain) to 120×120 nm (the transition to a chaotic state completed). Simulations were carried out using our MicroMagus simulation package [8] for which we have written an additional module to include the spin injection in form of the Slonczewski torque $\Gamma = (a_J/M_S) \cdot [\mathbf{M} \times [\mathbf{M} \times \mathbf{S}]]$ ($\mathbf{S}$ is the spin polarization direction of the current through a layer). All results presented here were obtained for $\mathbf{S}$ and the external field $H_{\text{ext}} = 1000$ Oe both directed along the $0x$-axis and the spin current strength $a_J = 0.4 M_S$. The Oersted field was not included to make the system as similar as possible to that studied in [7]. The lateral mesh size mostly used was 2×2 nm. We have checked that all results were nearly independent on the discretization: doubling the number of discretization cells along each side never led to any qualitative changes of the magnetization configurations or power spectra (see below) and resulted in the shift of the spectral peak positions up to maximum 5 %.

The coarse trend describing the system behaviour when the lateral size b of the square element increases is relatively simple. For very small sizes (up to $b \approx 20$ nm) the element behaves itself as a macrospin (first row in Fig. 1) - the magnetization configuration for any time remains almost collinear. The 3D trajectory of the average magnetization

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$\mathbf{m}^{\rm av}$ shows a well known 'out-of-plane' precession which was predicted by both single-spin [6] and finite-element micromagnetic simulations [7]. The power spectrum of $m_x^{\rm av}$-oscillations in this case exhibits a very sharp peak at the precession frequency $f_{\rm prec}$ and much smaller peaks with rapidly decreasing amplitudes by $2f_{\rm prec}$, $3f_{\rm prec}$, etc. due to the slightly non-sinusoidal character of the precession.

When the element size is increased, the maximal precession angle also increases until the 'out-of-plane' precession trajectory touches the element $(0xy)$ plane (this takes place for $b = 32 \pm 1$ nm) and the transition to the 'butterfly' trajectory occurs. The magnetization structure exhibits clear deviations from the single-domain one: the maximal angle between magnetization vectors at various element points can exceed 45^o. Although the total oscillation period *increases* (the limiting cycle is now much longer than for a simple quasi-elliptical 'out-of-plane' precession), the spectrum peak of $m_x^{\rm av}$-oscillations moves towards *higher* frequencies because one cycle includes now *two* oscillation periods for $m_x^{\rm av}$ (second row in Fig. 1). The weak satellites of the main spectral lines are due to a slow back-and-forth displacement of the $\mathbf{m}^{\rm av}$-trajectory in the region near the $0xy$-plane which is due to the fact that the energy landscape there is nearly flat.

Further enlargement of the element leads to the formation of well defined domains with sharp domain walls between them. This, in turn, results in the transition to the *quasiperiodic* $\mathbf{m}^{\rm av}$-trajectory, which now fills a bended torus (third row for $b = 60$ nm in Fig. 1). The co-existence of several domains explains the broadening of the corresponding spectral line and the shift of the $m_x^{\rm av}(t)$-dependence towards higher $m_x^{\rm av}$-values - due to the external field in the positive $0x$-direction.

For still larger element sizes the transition to the *chaotic* behaviour finally occurs (see example for $b = 80$ nm in the fourth row in Fig. 1). The magnetization trajectory completely fills in an area near the pole $m_x = 1$, $m_y = m_z = 0$. The size of the filled area decreases with the increasing element size due to the same aligning effect of the external field. The spectral power is gradually transferred to very low frequencies, because significant changes of the average $m_x^{\rm av}$-value at a time scale much larger than the (still visible) oscillation period occur due to the chaotic domain structure of the nanoelement.

Simulations for several intermediate system sizes reveal that the transition from the single-domain to the chaotic behaviour described above contains a very interesting intermediate stage (Fig. 2). First of all we note that although the qualitative behaviour of the *average* magnetization for the sizes, e.g., $b = 40$ nm and $b = 52$ nm is the same - the 3D $\mathbf{m}^{\rm av}$-trajectory of the 'butterfly' type is observed and corresponding spectra are quite similar (the first and the last rows in Fig. 2), magnetization patterns during the precession are very different (Fig. 3).

For the smaller size $b = 40$ nm the magnetization direction inside the element remains roughly the same (maximal angle between $\mathbf{m}(\mathbf{r})$-vectors at various element points is $\sim 50^o$). Pairs of semicircular 'quasidomains' are formed near the opposite element sides during the precession (Fig. 3, upper graph for $b = 40$ nm) and transitions between the areas with different $\mathbf{m}(\mathbf{r})$-directions are very smooth.

In contrast, magnetization configuration for the $b = 52$ nm element exhibits two sharply defined domains with nearly opposite magnetization directions (Fig. 3, lower picture for $b = 52$ nm). The corresponding domain wall moves up and down during the steady-state precession resulting in the overall change of $\mathbf{m}^{\rm av}$. The physical reason for this difference probably is that due to the larger element size it is energetically more favourable to form two relatively well defined and nearly homogeneously magnetized domains than to keep the configuration where the magnetization direction varies smoothly along the whole nanoelement.

The transition between the two 'butterfly'-regimes described above occurs in a very interesting way. When the size is increased above $b = 40$ nm, the limiting cycle broadens into a band of trajectories which width is maximal in the already mentioned critical region near the $0xy$-plane (see 3D-picture for $b = 46$ nm in the middle of the second row in Fig. 2); this also results in the shift and the broadening of the main spectral line. By further (very small !) size increase up to $b = 48$ nm the continuous band of trajectories splits itself and all trajectories collapse into 3 limiting subcycles (third row in Fig. 2). A peak at a frequency much lower than that of the 'normal' precession corresponding to the complete motion cycle over all these three subcycles appears in the spectrum together with its harmonics. And finally, the size increase up to $b = 52$ nm let all the subcycles collapse into a single 'butterfly'-type limiting cycle (see above).

The detailed discussion of these results will be given elsewhere. Here we only mention that, according to our simulations, the transitions described above are shifted, as expected, towards higher lateral sizes when the exchange constant or the element thickness is increased. Furthermore, the smoothing of the square corners also stabilizes to some extend the homogeneous magnetization configuration. However, for all studied values of the exchange constants (up to $A = 4 \times 10^{-6}$ erg/cm), all thicknesses up to $d = 5$ nm and all smoothing radii (up to the circular shape) the element with the lateral size $b = 120$ nm was always at least in a multi-domain state. We also note that an inclusion of the Oersted field shifts all transitions towards *smaller* sizes because this field is strongly inhomogeneous.

Some brief comments concerning theoretical interpretation of our results and their relation to experimental observa-

tions are in order. First we note that a theoretical analysis of our simulation data obviously requires the application of the non-linear dynamics formalism [9]. For example, qualitative changes of the magnetization trajectories shown in Fig. 2 strongly resemble pictures of processes known as 'period multiplication' bifurcations; time-dependent trajectories for, e.g., $b = 48$ nm are very similar to those known from the Lorentz attractor studies [9]; and the overall behaviour of the system when the size is increased from the smallest $b = 20$ nm to the largest one $b = 120$ nm obviously represents a nice example of the transition from a regular to chaotic behaviour when one of the system parameters (the nanoelement size in our case) is varied [9].

However, we point out that the corresponding rigorous analysis using the non-linear dynamics terms will be extremely difficult. The reason is that our system is actually a $2N$-dimensional one, where $N = N_x \times N_y$ is the total number of the discretization cells. This means that trajectories of the *average* magnetization shown in Fig. 1 and 2 are *not* the trajectories in the system phase space which are required for the analysis with the non-linear dynamics methods, but merely the trajectories of a certain low-dimensional (3D) functionals defined on this phase space. In the same fashion, the analysis of the system behaviour in terms of Ljapunov exponents would require the (very precise !) determination of the eigenvalues of the corresponding $2N \times 2N$ matrices which is a tedious computational task, leaving apart a rather non-trivial interpretation problem of the resulting sets of $2N$ eigenvalues. For these reasons we believe that such studies should be carried out only if one expects from them new deep insights into the physics of the process beyond those which can be obtained by studying the standard physical characteristics like discussed above.

Results like those shown here may be in principle compared *directly* with the experimental data - especially since high-quality measurements of the spin-transfer induced microwave oscillation spectra for various systems are available [4]. However, our simulations demonstrate that for the nanoelement sizes commonly used in such experiments, the nature of the precessional state (which is necessary to ensure the very existence of the non-decaying m/w oscillations) is very sensitive to the element size. Furthermore (corresponding results will be reported elsewhere) the shape of the element and its polycrystalline structure (for materials with significant magnetocrystalline anisotropy) also play an important role. Apart from the evident consequence that simulations of a spin-torque induced precession of nanoelements with experimentally interesting sizes in a macrospin approximation are not very helpful, our results show that a *precise* determination of all mentioned above physical parameters of the experimentally studied system is required to enable a meaningful comparison with micromagnetic simulations. The latter step is, in turn, necessary for a real understanding of physical processes underlying the magnetization dynamics in the presence of a spin-polarized current.

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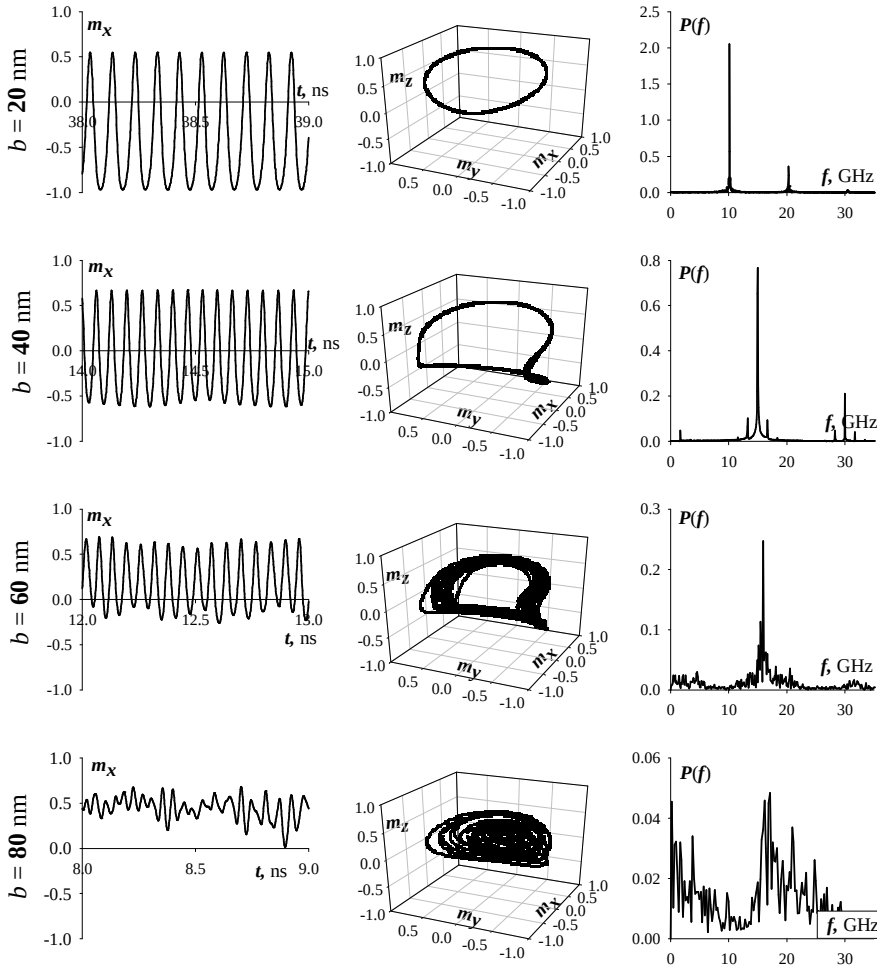

FIG. 1: Time-dependencies of the x-projection of the average element magnetization $m_x^{\rm av}$ (first column), 3D trajectories of $\mathbf{m}^{\rm av}$ (second column) and spectra of $m_x^{\rm av}$ (third column) for various element sizes as indicated on the left.

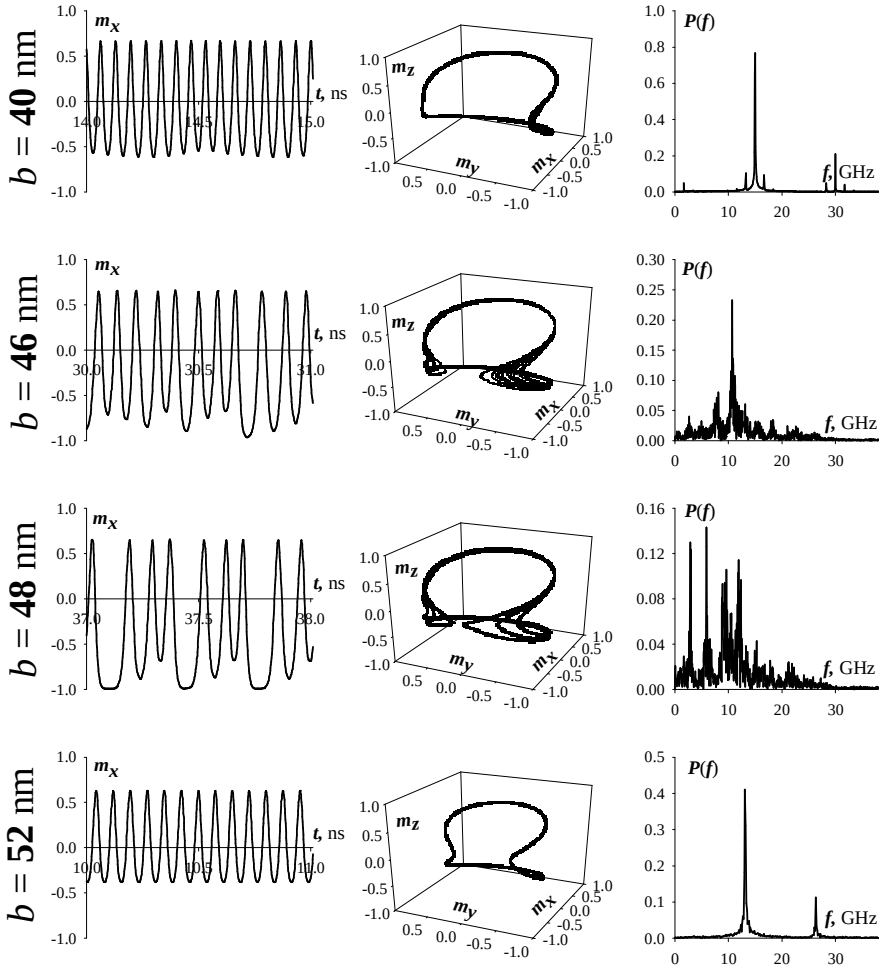

FIG. 2: The same as in Fig. 1 for element sizes from $b = 40$ nm to $b = 52$ nm

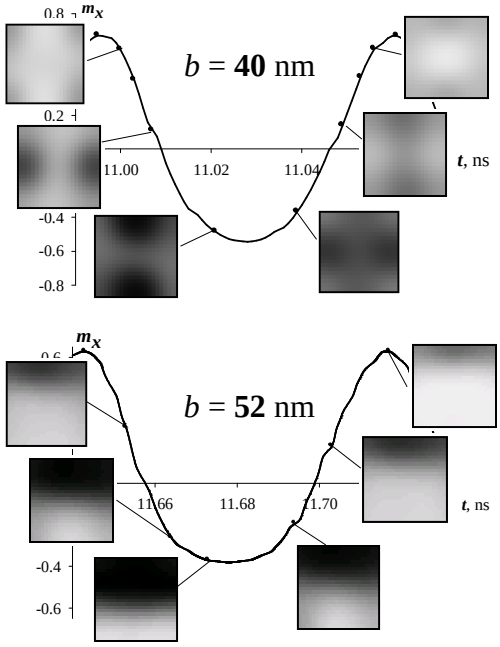

FIG. 3: Comparison of magnetization patterns during the steady-state precession for elements with the sides $b = 40$ nm and $b = 52$ nm